Embankment basal stability analysis using shear strength reduction finite element method

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ABSTRACT: The stability evaluation of the slope commonly uses limit equilibrium methods, for example, Fellenius' method in Japan. The limit equilibrium methods have two limitations. (1) It is necessary to divide the sliding mass into slices, and to set up some additional equations to make the problem statically determinate. (2) It is necessary to search sliding surface with a minimum safety factor. Shear strength reduction finite element method (SSR-FEM) can solve the limitations of the limit equilibrium methods. This paper inspects superiority of the slope stability analysis by SSR-FEM. And, as a model case, the failure of base ground of the embankment is considered. Comparing the calculated results of SSR-FEM with limit equilibrium method, we point out the problems inherent in limit equilibrium method for the model case when the slip surface is assumed to be circle. We propose an approach to use SSR-FEM for slope stability analysis in practical design.

1 INTRODUCTION

The stability evaluation of the slope commonly uses limit equilibrium methods, for example, Fellenius' method, in most design codes and standards in Japan. The limit equilibrium methods have two limitations.(1) It is necessary to divide the sliding mass into slices, and to set up some additional equations to make the problem statically determinate.(2) It is necessary to search sliding surface with a minimum safety factor. Shear strength reduction finite element method (SSR-FEM) can solve the limitations of the limit equilibrium methods.

SSR-FEM can obtain the safety factor and slip surface without the analyzer to assume any particular shape about the slip surface. SSR-FEM begins to be used to analyze the slope stability though it is not commonly in the design. The limit equilibrium methods are still used in almost cases. It is because the limit equilibrium methods are simple. However, the limit equilibrium methods are not fully verified for the following items.(1) Whether it is correct to assume that the slip surface is one circular arc.(2) How wide is the range of the grid for the center of slip circular arc to search the minimum safety factor.

Then, embankment base failure of the support ground of the fill was taken up as a model case in this report. When the support ground consists of clayey soil layer and bearing stratum, the thickness of the clayey soil layer is changed, and the slip surface are analyzed using SSR-FEM. Using these obtained slip surfaces, we arranges the extent that the slip surface become a circular arc or a non-circular arc. It is the main one of objectives t the stability examination that uses the SSR-FEM from the stability examination by the limit equilibrium method in this report.

2 SSR-FEM FOR STABILITY ANALYSIS

2.1 *Basic concept of SSR-FEM*

In the finite element method with shear strength reduction technique(SSR-FFM),a non-associated elasto-plastic constitutive law is adopted, where the Mohr-Coulomb yield criterion is used to define the yield function.

$$f = \frac{\sigma_1 - \sigma_2}{2} - c'\cos\phi' - \frac{\sigma_1 - \sigma_2}{2}\sin\phi' \quad (1)$$

and the Drucker-Prager criterion to define the plastic potential.

$$g = -\alpha I_1 + \sqrt{J_2} - \kappa \quad (2)$$

Where

$$\alpha = \frac{\tan\psi}{\sqrt{9 + 12\tan^2\psi}}, \kappa = \frac{3c'}{\sqrt{9 + 12\tan^2\psi}} \quad (3)$$

In Equation 1 and 2, c', ϕ', and ψ are the effective cohesion, friction angle, and dilatancy angle, resperctively. I_1 and J_2 are the first invariant of the effective stress, and the second invariants of the deviatoric stress, respectively. σ_1 and σ_3 are the majar and minor principal effective stress, respectivety.

The global safety factor of slope in SSRFEM identical to the one in limit equilibrium methods. The reduced strength parameters c'_F and ϕ'_F are defined by

$$c'_F = c'/F \, , \phi'_F = \tan^{-1}(\tan\phi'/F) \qquad (4)$$

In SSRFEM, firstly, the initial stresses in slope are computed using the elastic finite element analysis. The vector of externally nodal forces consists of three parts: (1) surface force; (2) body force(total unit weight of soils); and (3) pore water pressure. Secondly, stresses and strains are calculated by the elasto-plastic finite element analysis, where the reduced shear strength criterion. The shear strength reduction factor F is initially selected to be so small, for example 0.01, that the shear strength is large enough to keep the slope in elastic stage. Stresses at some Gaussian points reach the yielding condition with the shear strength reduction factor F in Equation 3 increased gradually. When the stress at anyone Gaussian point reaches the yielding condition, the increment of the shear strength reduction factor will make stresses at more Gaussian points reach the yielding condition because of the residual force induced by the decrease in the shear strength soils.

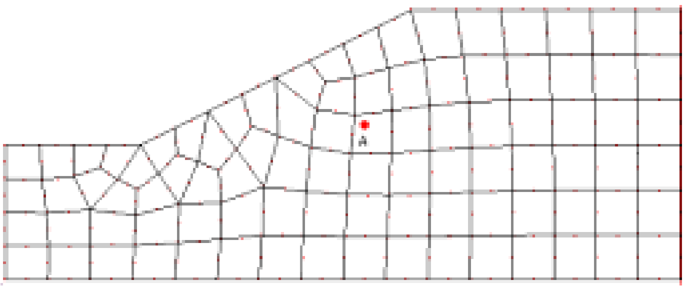

Figure 1 A point

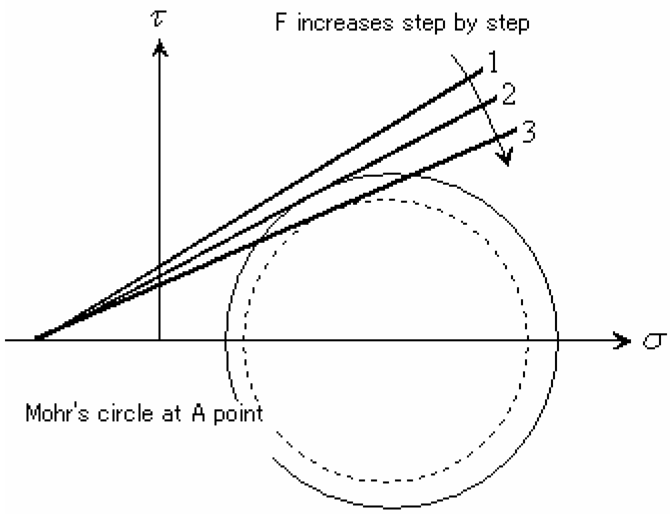

Figure 2. Mohr's circle at A point.

The shear strength reduction factor F increases incrementally until the global failure of the slope reaches, which means that the finite element calculation diverges under a physically real convergence criterion. The lowest factor of safety of slope lies between the shear strength reduction factor F at which the iteration limit is reached, and the immediately previous one. The procedure described hereby can predict the factor of safety within one loop, and can be easily implemented in a computing code.

One of the main advantages of SSRFEM is that the safety factor emerges naturally from the analysis without the user having to commit to any particular from of the mechanism a priori. When the slope stability is evaluated with the effective stress method, the pore water pressure is usually predicted with the finite element seepage analysis or Biot's consolidation theory. If the same finite element mesh is used for the seepage or consolidation analysis and SSRFEM, the water pressure, predicted in the seepage or consolidation analysis, can be directly used in SSRFEM. This can simplify the slope stability analysis, and consider more accurately the influence of the seepage force.

2.2 *Predominance of SSRFEM*

This example was firstly reported by Arai and Tagyo and concerns a layered slope (Figure 3), where a layer of low resistance is interposed between two layers of higher shear strength. The material properties of various layers are listed in Table 1. Arai and Tagyo used the conjugate gradient method to search for the critical slip surface and used simplified Janbu's method to calculate the safety factor, and obtained the lowest safety factor of 0.405. This problem has also been examined by Sridevi and Deep using the random-search technique RST-2, by Greco using pattern search and Monte Carlo method, and by Malkawi et al. using Monte Carlo method (random walking). The slip surfaces located by various investigators are of significant difference. In this study, the problem is analyzed and Table 2 summarizes the results. Figure.4 shows the slip surface determined by SSRFEM. It is clear that the slip surface by SSRFEM is closer to that located by Greco.

Table 1 Material Properties for Example

Layer	ϕ'(Deg)	c'(kN/m2)	γ (kN/m2)
1	12	29.4	18.82
2	5	9.8	18.82
3	40	294.0	18.82

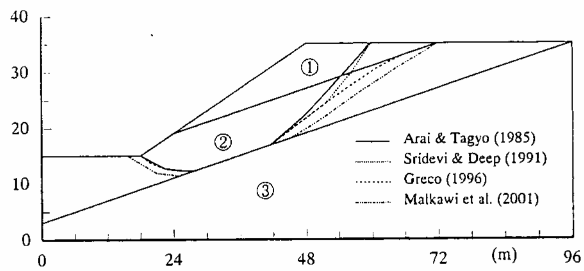

Figure 3. Slope in example.

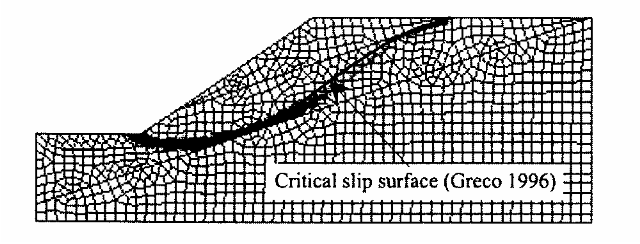

Figure 4. Slip surface by SSRFEM for example.

Table 2 Minimum Safety Factors for Example

Method for safety factor	Method for slip surface	Safety factor
Arai and Tagyo (1985)		
Simplified Janbu	Conjugate gradient	0.405, 0.430※
Sridevi and Deep (1992)		
Simplified Janbu	RST-2	0.401, 0.423※
Greco (1996)		
Spencer	Pattern search	0.388
Spencer	Monte Carlo	0.388
Malkawai et al.(2001)		
Spencer	Monte Carlo	0.401
Rocscience Inc. (2002)		
Spencer	Random search	0.401
Simplified Janbu	Random search	0.410, 0.434※
Kim et Al. (2002)		
Spencer	Random search	0.44
Lower-bound	Automatic	0.40
Upper-bound	Automatic	0.45
Cai et Al. (2003)		
SSRFEM ($\psi = 0$)	Automatic	0.417
SSRFEM ($\psi = \phi$)	Automatic	0.423

※ corrected the safety factor calculated by simplified Janbu

3 APPLICATION OF SSR-FEM

3.1 *Embankment basal failure model*

This paper compared limit equilibrium methods (Bishop's simplified method) and SSR-FEM($\psi = \phi$) about embankment base failure of the support ground of the fill. Figure 5 shows the model embankment. The embankment was assumed 20.0m in width of levee crown, 10.0m in height **(h1)**. In two levels of strata, the upper layer was soft, the lower layer was strong. The thickness **(h2)** of the upper layer was changed from 2.0m, 4.0m, 6.0m, 8.0m, to 10.0m, to compare the results of both analytical methods.

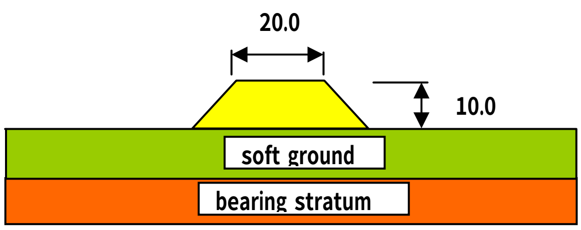

Figure 5. Embankment basal destruction model.

Table 3 Material Properties for Test Model

Layer	ϕ (Deg)	c(kN/m2)
embankment	35.0	0.00
soft ground	0.0	35.0
bearing stratum	0.0	100.0

Material properties is show in Table 3. The necessary matrix of a plasticity calculation was defined using Young's modulus and Poisson's ratio. The influence of these coefficients before the failure is great, but the influence on the total safety factor is very small (Zienkiewicz et al.1975, Griffiths and Lane 1999).

Therefore, we assumed Young's modulus of $2x10^4$kN/m^2, Poisson's ratio of 0.3 regardless of the soils. As for the embankment, φ =35.0 degrees, the adhesive strength of the soft layer assumed 35kN/m^2, the adhesive strength of the bearing stratum 100kN/m^2.

3.2 *Analysis result*

3.2.1 *h2 = 2.0m*

Figure 6(a) shows an analysis result of h2=2.0m. In SSR-FEM, the slip surface occurred in soft layer base and was a non-circular arc. In contrast, circular slip was searched by the limit equilibrium method located at the embankment slope. The sliding mechanics was different. As for the safety factor, SSR-FEM gave 1.24, and the limit equilibrium method gave 1.21.

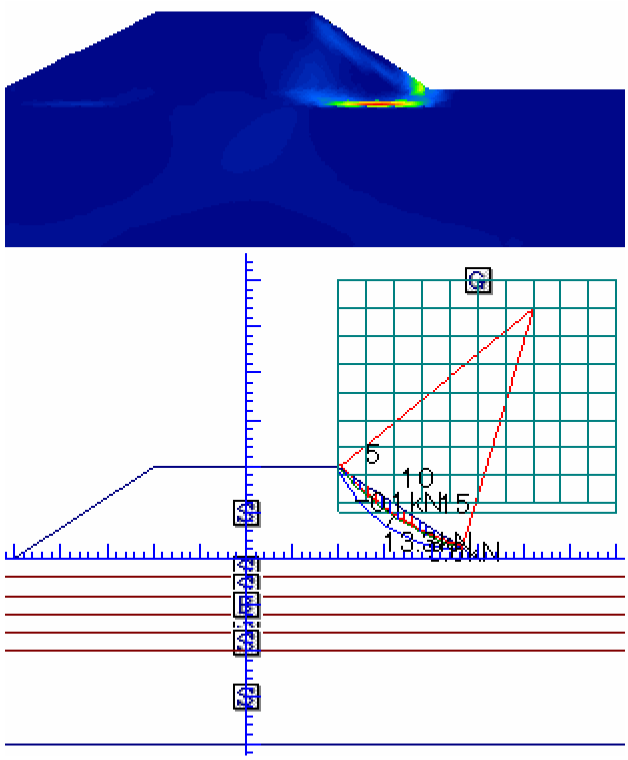

Figure 6(a). Result of SSRFEM and LEM h2=2.0m

3.2.2 *h2 = 4.0m*

Figure 6(b) shows an analysis result of h2=4.0m. In SSR-FEM and limit equilibrium method, the slip surface occurs in soft layer base and become a non-circular arc. As for the safety factor, SSR-FEM gave 1.16, and limit equilibrium method gave 1.22, and some differences occurred between the two methods.

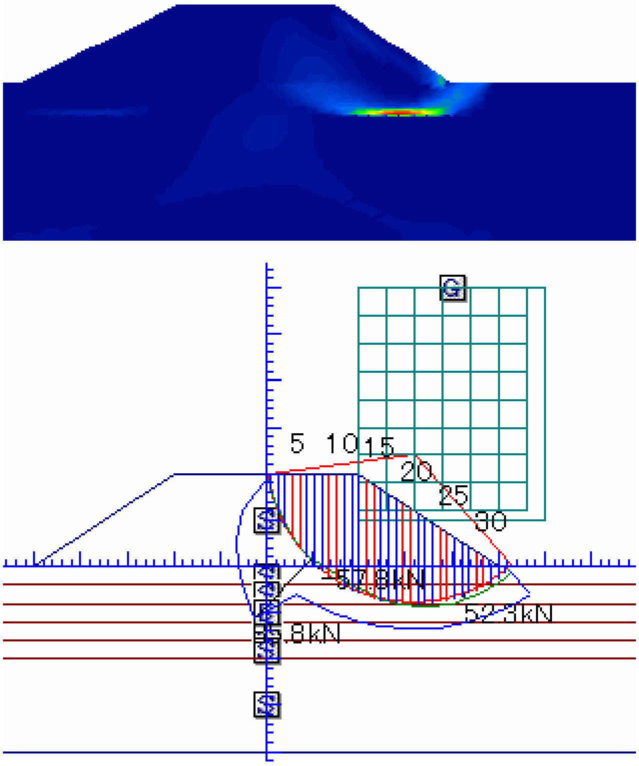

Figure 6(b). Result of SSRFEM and LEM h2=4.0m

3.2.3 *h2 = 6.0m*

Figure 6(c) shows an analysis result of h2=6.0m. In SSR-FEM and limit equilibrium method, the slip surface occurs in soft layer base. As for the safety factor, SSR-FEM gave 1.16, and LEM gave 1.22.

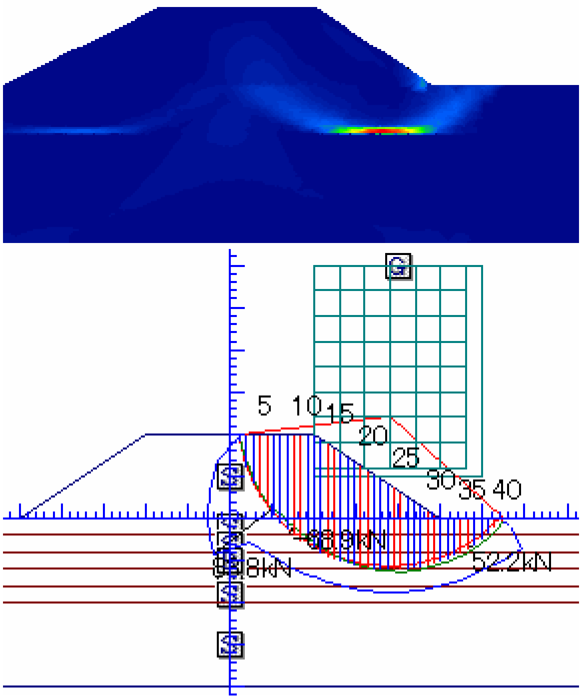

Figure 6(c). Result of SSR-FEM and LEM (h2=6.0m)

3.2.4 *h2 = 8.0m*

Figure 6(d) shows an analysis result of h2=6.0m. In SSR-FEM and limit equilibrium method, the slip surface occurs in soft layer base. As for the safety factor is both 1.10.

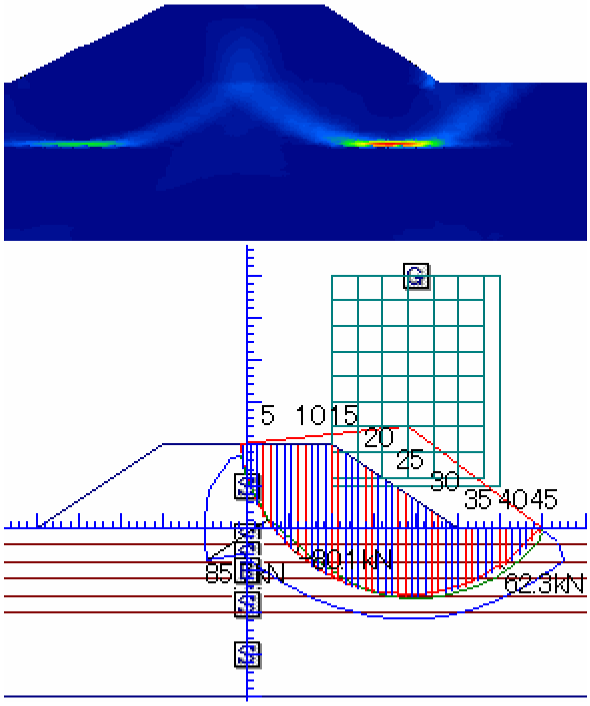

Figure 6(d). Result of SSR-FEM and LEM (h2=8.0m)

3.2.5 *h2 = 10.0m*

Figure 6(e) shows an analysis result of h2=10.0m. In SSR-FEM and limit equilibrium method, the slip surface occurs in soft layer base. As for the safety factor is both 1.08.

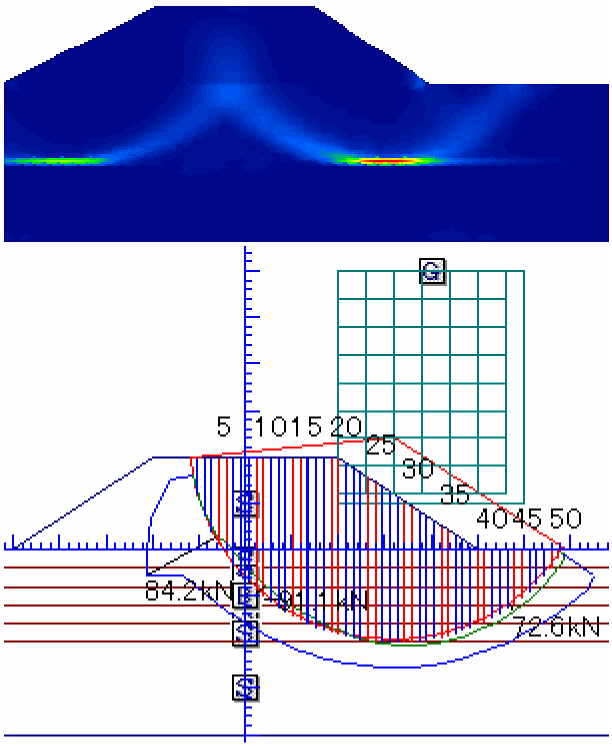

Figure 6(e). Result of SSR-FEM and LEM (h2=10.0m)

3.3 *The comparison of the analysis result*

Table 4 shows the results of both methods. Figure 7 shows the results between the safety factor and the ratio of thickness (h2) of the soft layer and embankment height (h1=10.0m).

Table 4. An analysis result table every layer thickness

h2(m)	2.00	4.00	6.00	8.00	10.00
h2/h1	0.20	0.40	0.60	0.80	1.00
SSR	1.211	1.165	1.132	1.097	1.075
LEM	1.240	1.220	1.130	1.100	1.080

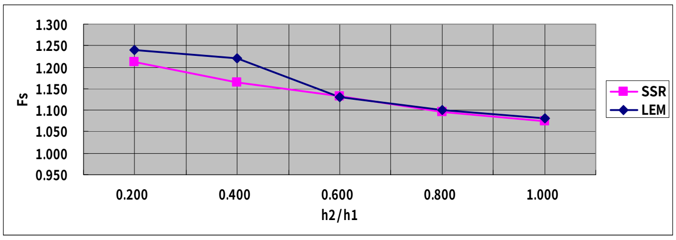

Figure7. A figure of comparison

Figure shows that both analysis results consistent with each other when the thickness of the soft layer is as above 6.0m (h2/h1≧0.6).

However, in the situation that a soft layer is thin, a difference occurs in SSRFEM and limit equilibrium method.

When a soft layer is very thin h2=2.0m (h2/h1=0.2), in SSR-FEM, bottom failure occurs. On the other hand, embankment slope failure occurs by limit equilibrium method. The calculated failure mechanics was different with each other.

Slip surface in the straight line occurs to go along the soft layer lower part from a result of SSR-FEM. In other words this case becomes the non-circular arc.

The limit equilibrium method assumes that it is circular arc sliding. As a result, it is thought that a safety factor searched the lowest circular arc sliding. However, the limit equilibrium method was not able to search the smallest safety factor and the sliding shape, because a safety factor of SSR-FEM is 1.21, and a safety factor of the limit equilibrium method is 1.24.

In h2=4.0m, embankment base failure occurs both in SSRFEM and limit equilibrium method. The slip surface occurs in soft layer base and become a non-circular arc. As for the safety factor, the latter was 1.16 former 1.22. Among both, some differences occur. In h2=6.0m are regarded as non-circular arc about the sliding shape in SSR-FEM about 8.0, 10.0m as follows. But, the ratio that the tangent holds shrinks as a soft layer thickens. Therefore, for a safety factor, is approximately equal.

Table 5 shows the result that measured the length of the straight line of the non-circular arc in SSR-FEM.

Table 5. Length of the straight line every layer thickness

h2(m)	2.00	4.00	6.00	8.00	10.00
L1(m)	6.3	6.3	7.0	7.5	7.5
L1/h2	3.15	1.57	1.16	0.93	0.75

According to the list, the length of the straight line gets longer so that the thickness of the soft layer increases, but the ratio for the layer thickness understands a thing becoming small. In other words the glide plane suffers from the shape that is near to an arc so that a soft layer is thick.

Figure 8. A non-arc by limit equilibrium method slips.

Figure 8 is a result of non-arc sliding by the limit equilibrium method that considered a straight line calculated by SSR-FEM about layer thickness 4.0m. It was 1.17, and the safety factor almost equal to that of SSR-FEM.

4 CONCLUSION

By this report, we compared an analysis result of SSR-FEM with the limit equilibrium method. With that in mind, we wanted to grasp such soil layer condition that sliding surface became the non-arc from the analysis result of SSR-FEM.

It is necessary to give the center of slip circular act by the judgment of the designer after having supposed sliding shape to be an arc by the limit equilibrium method. Therefore, the limit equilibrium method cannot cope with a non-arc even if the minimum safety factor was able to search the circular act.

SSR-FEM does not have to assume an examination condition, in a given condition, there is the big advantage that it is slippery, and can search shape to become the minimum safety factor.

When the thickness of the layer was thin, SSR-FEM could search a non-arc as sliding line associated with the minimum safety factor, and, in the example, the difference with the limit equilibrium method became great.

By this report, SSR-FEM analyses were performed to find the parameter which was ascertained whether the sliding shape was non-circular. The soft layer thickness was changed and a series of analysis was carried out.

Although the series of analyses was performed, a good parameter was not found to indicate the layer thickness of the soft layer under which non-arc sliding should occur. A future search should be performed.

At present, we advise that the sliding shape was calculated using SSR-FEM, the safety factor of the calculated slip surface was evaluated using limit equilibrium method wanted by design codes and standards.

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